Can EPR correlations be driven by and effective wormhole?*

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Abstract

We consider the two-particle wave function of an EPR system given by a two dimensional relativistic scalar field model. The Bohm-de Broglie interpretation is applied and the quantum potential is viewed as modifying the Minkowski geometry. In such a way singularities appear in the metric, opening the possibility, following Holland, of interpreting the EPR correlations as originated by a wormhole effective geometry, through which physical signals can propagate.

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A causal approach to the Einstein-Podolsky-Rosen (EPR) problem, i.e. a two-particle correlated system, is developed. We attack the problem from the point of view of quantum field theory considering the two-particle function for a scalar field and interpreting it according to the Bohm - de Broglie view. In this approach it is possible to interpret the quantum effects as modifying the geometry in such a way that the scalar particles see an effective geometry. For a two-dimensional static EPR model we are able to show that quantum effects introduces singularities in the metric, a key ingredient of a bridge construction or wormhole. Following a suggestion by Holland [1] this open the possibility of interpret the EPR correlations as driven by an effective wormhole[1].

The two-particle wave function of a scalar field, $\psi_2(\mathbf{x_1},\mathbf{x_2},\mathbf{t})$ satisfies(see for example [3] [4]):

$$\sum_{j=1}^{2}[(\partial^\mu\partial_\mu)_j+m^2]\psi_2(\vec{\mathbf{x}}^{(2)},t)=0 \tag{1}$$

where $\vec{\mathbf{x}}^{(n)}\equiv\{\mathbf{x_1},...\mathbf{x_n}\}$. Explicitly we have

$$[(\partial^\mu\partial_\mu)_1+m^2]\psi_2(\mathbf{x_1},\mathbf{x_2},\mathbf{t})+[(\partial^\mu\partial_\mu)_\mathbf{2}+\mathbf{m^2}]\psi_\mathbf{2}(\mathbf{x_1},\mathbf{x_2},\mathbf{t})=\mathbf{0}. \tag{2}$$

Substituting $\psi_2=R\exp(iS/\hbar)$ in Eq. (2) we obtain two equations, one of them for the real part and the other for the imaginary part. The first equation reads

$$\eta^{\mu_1\nu_1}\partial_{\mu_1}S\partial_{\nu_1}S+\eta^{\mu_2\nu_2}\partial_{\mu_2}S\partial_{\nu_2}S=2\mathcal{M}^2 \tag{3}$$

where

$$\mathcal{M}^2\equiv m^2\hbar^2(1-\frac{Q}{2m^2\hbar^2}) \tag{4}$$

with $Q\equiv Q_1+Q_2$ being $Q_1=-\hbar^2\frac{(\partial^\mu\partial_\mu)_1R}{R}$ and $Q_2=-\hbar^2\frac{(\partial^\mu\partial_\mu)_2R}{R}$. (4′)

The equation that comes from the imaginary part is

$$\eta^{\mu_1\nu_1}\partial_{\mu_1}(R^2\partial_{\nu_1}S)+\eta^{\mu_2\nu_2}\partial_{\mu_2}(R^2\partial_{\nu_2}S)=0 \tag{5}$$

which is a continuity equation.

[1] An extended version of this talk can be found in [2] where a non-tachyonic EPR model is studied

Following De Broglie [5] we rewrite the Hamilton-Jacobi equation (3) as

$$\frac{\eta^{\mu_1\nu_1}}{(1-\frac{Q}{2m\hbar^2})}\partial_{\mu_1}S\partial_{\nu_1}S+\frac{\eta^{\mu_2\nu_2}}{(1-\frac{Q}{2m\hbar^2})}\partial_{\mu_2}S\partial_{\nu_2}S=2m^2\hbar^2. \tag{6}$$

Here $\eta^{\mu\nu}$ is the Minkowski metric and we can interpret the quantum effects as realizing a conformal transformation of the metric in such a way that the effective metric is $g_{\mu\nu} = (1-\frac{Q}{2m^2\hbar^2})\eta_{\mu\nu}$. Now, following an approach by Alves (see [7]), we will see that for the static case it is possible to obtain a solution as an effective metric which comes from Eqs. (3) and (5). For the static case these equations are:

$$\eta^{11}\partial_{x_1}S\partial_{x_1}S+\eta^{11}\partial_{x_2}S\partial_{x_2}S=2m^2\hbar^2(1-\frac{Q}{2m\hbar^2}) \tag{7}$$

$$\partial_{x_1}(R^2\partial_{x_1}S)+\partial_{x_2}(R^2\partial_{x_2}S)=0 \tag{8}$$

We consider that our two-particle system satisfies the EPR condition $p_1 = -p_2$ which in the BdB interpretation, using the Bohm guidance equation $p = \partial_x S$, can be written as $\partial_{x_1}S = -\partial_{x_2}S$. Using this condition in Eq. (8) we have $\partial_{x_1}(R^2\partial_{x_1}S) = \partial_{x_2}(R^2\partial_{x_1}S)$ and this equation has the solution $R^2\frac{\partial S}{\partial x_1} = G(x_1+x_2)$ where G is an arbitrary (well behaved) function of x_1+x_2. Substituting in Eq.(7) we have

$$2m^2\hbar^2(1-\frac{Q}{2m\hbar^2})=2(\frac{G}{R^2})^2 \tag{9}$$

and using the expression (4') for the quantum potential, the last equation reads

$$8G^2+(\partial_{x_1}(R^2))^2-2R^2\partial^2_{x_1}R^2+(\partial_{x_2}(R^2))^2-2R^2\partial^2_{x_2}R^2-8m^2R^4=0. \tag{10}$$

A solution of this nonlinear equation is $R^4 = \frac{1}{2m^2}(C_1\sin(m(x_1+x_2)+C_2))$ provided an adequated function $G(x_1+x_2)$ which can be obtained from (10) by substituting the solution.

In order to interpret the effect of the quantum potential we can re-write Eq. (7) using (9) obtaining $m^2\frac{\eta^{11}}{(\frac{G}{R^2})^2}\partial_{x_1}S\partial_{x_1}S+m^2\frac{\eta^{11}}{(\frac{G}{R^2})^2}\partial_{x_2}S\partial_{x_2}S=2m^2$ that we write as

$$g^{11}\partial_{x_1}S\partial_{x_1}S+g^{11}\partial_{x_2}S\partial_{x_2}S=2m^2 \tag{11}$$

and then we see that the quantum potential was "'absorbed"' in the new metric g_{11} which is:

$$g_{11} = \frac{1}{g^{11}} = \frac{\eta_{11}}{m^2}(\frac{G}{R^2})^2 = \frac{-\frac{C_1^2}{16m^2} + \frac{3C_1^2}{16m^2}\sin^2(m(x_1+x_2)+C_2)}{\frac{C_1}{2m^2}\sin(m(x_1+x_2)+C_2)}. \quad (12)$$

We can see that this metric is singular at the zeroes of the denominator in (12)and this is characteristic of a two dimensional black hole solution (see [6] [7]). Then our two-particle system "see" an effective metric with singularities, a fundamental component of a whormhole[8]. This open the possibility, following Holland [1], of interpret the EPR correlations of the entangled particles as driven by an effective wormhole. Obviously a more realistic (i.e. four dimensional) and more sophisticated model (i.e. including the spin of the particles) must be studied. [2]

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[2] It is interesting to note that a wormhole coming from a (Euclidean) conformally flat metric with singularities was shown by Hawking [9]. Consider the metric:

$$ds^2 = \Omega^2 dx^2 \quad (13)$$

with

$$\Omega^2 = 1 + \frac{b^2}{(x-x_0)^2}. \quad (14)$$

This looks like a metric with a singularity at x_0. However, the divergence of the conformal factor can be though as the space opening out to another asymptotically flat region connected with the first one by mean a wormhole of size $2b$.

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